

1. A tank initially contains 1000 kg of brine containing 10% salt by mass. An inlet stream of brine containing 20% salt by mass flows into the tank at a rate of 20 kg/min. The mixture in the tank is kept uniform by stirring. Brine is removed from the tank via an outlet pipe at a rate of 10 kg/min. Find the time when the amount of salt in the tank is 200 kg. [20 points]
2. Fluid flows between two parallel plates, a distance h apart. The upper plate moves at velocity v_0 while the lower plate is stationary. For what value of pressure gradient will the shear stress at the lower plate be zero. [15 points]
3. Determine the velocity profile in a fluid situated between two coaxial rotating cylinders. Let the inner cylinder have radius R_1 and angular velocity Ω_1 . Let the outer cylinder have radius R_2 and angular velocity Ω_2 . [15 points]
4. Interpret the **physical meanings** of the following dimensionless groups: (a) Froude number (b) Péclet number (c) Schmidt number (d) Brinkman number. [total 20 points; 5 points for each item]
5. Estimate the time t_f required for a liquid drop of radius R to freeze completely, if the drop is initially at its melting temperature T_0 and the surrounding air is at T_∞ . Heat is lost from the drop to the surrounding air according to the Newton's law of cooling, with a constant heat-transfer coefficient h . Assume no volume change in the solidification process. Solve the problem by using a **quasi-steady-state** method:
(a) First solve the steady-state heat conduction problem in the solid phase in the region between $r = R_f$ (the liquid-solid interface) and $r = R$ (the solid-air interface). Let k be the thermal conductivity of the solid phase. Then find the radial heat flow Q across the spherical surface at $r = R$. [15 points]
(b) Then write down an unsteady-state energy balance, by equating the heat liberation at $r = R_f(t)$ resulting from the freezing of the solid to the heat flow Q across the spherical surface at $r = R$. Integrating the resulting separable, first-order differential equation between the limits 0 and R , to obtain the time that it takes for the drop to solidify. Let $\Delta\hat{H}_f$ be the latent heat of freezing per unit mass. [15 points]