

1. A potato is put into a pot of water where the water temperature remains constant, A . The temperature of the potato, T , varies with the time, t , in the water. The experiments found that the rate of the change of the temperature of the potato (dT/dt) is proportional (a constant, K) to the difference between the temperature of the potato and the water ($T-A$).
 - (a) (5 points) Express the mathematical model which serves the situation described above,
 - (b) (5 points) if $K=1$ and $A=0$ (arbitrary unit), find the general solutions of the equation,
 - (c) (5 points) if $K=1$, $A=0$ and $T(0) = 100$, find the solution, and
 - (d) (5 points) draw a graph (T v.s t) representing the solution.
2. (15 points) Solve: $2xy' - 3y = 9x^3$, by the method of integrating factors.
3. (15 points) Using Laplace transform, find the solution of $y'' - y = e^{-t}$ for which $y(0)=1$, $y'(0)=0$.
4. (20 points) A single parameter function is defined as follows: $f(x) = x + x^2$ in the region of $-1 < x < 1$. Find the Fourier series of $f(x)$.
5. A fluid velocity is given $\mathbf{v} = (x^2 + y^2x)\delta_x + (y^2 + x^2y)\delta_y$, where δ_x and δ_y are the unit vectors in the x and y axes, respectively.
 - (a) (10 points) Is \mathbf{v} an incompressible flow field?
 - (b) (10 points) What is the velocity potential of the flow field?
6. (10 points) A matrix is shown below $\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$. Evaluate eigenvalues and eigenvectors of the matrix.