

1. A potato is put into a pot of water where the water temperature remains constant,  $A$ . The temperature of the potato,  $T$ , varies with the time,  $t$ , in the water. The experiments found that the rate of the change of the temperature of the potato ( $dT/dt$ ) is proportional (a constant,  $K$ ) to the difference between the temperature of the potato and the water ( $T-A$ ).
  - (a) (5 points) Express the mathematical model which serves the situation described above,
  - (b) (5 points) if  $K=1$  and  $A=0$  (arbitrary unit), find the general solutions of the equation,
  - (c) (5 points) if  $K=1$ ,  $A=0$  and  $T(0) = 100$ , find the solution, and
  - (d) (5 points) draw a graph ( $T$  v.s  $t$ ) representing the solution.
2. (15 points) Solve:  $2xy' - 3y = 9x^3$ , by the method of integrating factors.
3. (15 points) Using Laplace transform, find the solution of  $y'' - y = e^{-t}$  for which  $y(0)=1$ ,  $y'(0)=0$ .
4. (20 points) A single parameter function is defined as follows:  $f(x) = x + x^2$  in the region of  $-1 < x < 1$ . Find the Fourier series of  $f(x)$ .
5. A fluid velocity is given  $\mathbf{v} = (x^2 + y^2x)\delta_x + (y^2 + x^2y)\delta_y$ , where  $\delta_x$  and  $\delta_y$  are the unit vectors in the  $x$  and  $y$  axes, respectively.
  - (a) (10 points) Is  $\mathbf{v}$  an incompressible flow field?
  - (b) (10 points) What is the velocity potential of the flow field?
6. (10 points) A matrix is shown below  $\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ . Evaluate eigenvalues and eigenvectors of the matrix.