1. Determine constants, a, b, and c, so that the function $y_0(t) = a$ and $y_1(t) = b + c\,t$ form an orthonormal set on the interval $0 \le t \le 1$. (12%)

2. Solve $\left(x^2 + y^2 + x\right)dx + xydy = 0$. Whether the equation is exact or not. Find an integrating factor and the solution. (12%)

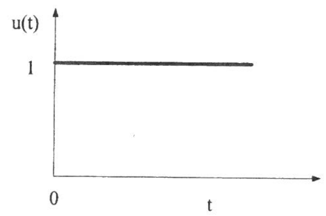
3. Find the eigen-values and the solution of the differential equation
$y'' - 3y' + 2y = e^{5t}, y(0) = \frac{13}{12}, y'(0) = \frac{5}{12}$ (12%)

4. Solve the differential equations using Laplace transforms

$$\frac{dx}{dt} + x(t) = u(t), x(0) = 0$$

$$2\frac{dy}{dt} + y(t) = x(t), y(0) = 0$$

where the input function $u(t)$ is the unit step function as below. Find the solutions x(t) and y(t). Explain whether the problem is stable or not. (14%)



5. Calculate the inverse of the following matrix by the Causs-Jordan elimination. (10%)

$$\begin{bmatrix} 1.5 & -1.5 & 0.5 \\ -1.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \end{bmatrix}$$

6. Assuming sufficient differentiability, show $\nabla\bullet(\mathbf{u}\times\mathbf{v}) = \mathbf{v}\bullet(\nabla\times\mathbf{u}) - \mathbf{u}\bullet(\nabla\times\mathbf{v})$, where **u** and **v** are vectors. (10%)

7. The one-dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}.$$

The two boundary conditions are $u(0, t) = 0$ and $u(L, t) = 0$ for all t. The two initial conditions are

$$u(x, 0) = f(x)$$

and

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)$$

i) Obtain two ordinary differential equations by applying the method of separating variables. (10%)

ii) Determine solutions of those two equations that satisfy the boundary conditions. (10%)

iii) Solve the entire problem that satisfy the initial conditions. (10%)