1. (20 points) Classify the following differential equations: state their order, degree and linearity, and indicate the independent variable(s) and dependent variable(s) of the equations. DO NOT attempt to solve them. (four points for each question)

   (a) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 10\sin x$

   (b) $\frac{dQ}{dt} = e^Q - 1$

   (c) $\cos x \left(\frac{dy}{dx}\right)^6 + \sin x \left(\frac{d^2y}{dx^2}\right)^3 = 0$

   (d) $\left(\frac{d^5x}{dt^5}\right)^2 = x$

   (e) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

2. (15 points) Homogeneous ordinary equation. Solve $(x^2+3y^2)dx - 2xy\,dy = 0$

3. (15 points) Laplace transform. Find the solution of $y'' - 3y' + 2y = 12\,e^{-2t}$ for which $y(0)=2, y'(0)=6$

4. (15 points) Evaluate the eigenvalues and eigenvectors of the matrix shown below.
$$\mathbf{A} = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

5. (15 points) C is the triangle whose vertices are $(0, 0)$, $(\pi/2, 0)$, and $(\pi/2, 1)$. Evaluate
$$\oint_C (y - \sin x)dx + \cos x\,dy$$

6. (20 points) Based on the Fourier integral, derive the explicit expression of $F(w)$ from the following equation.
$$e^{-x^2} = \int_0^\infty F(w)\cos wx\,dw$$