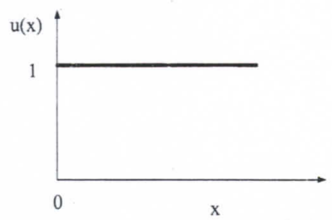
1. Find the particular solution of $\frac{dy}{dx}=\frac{x^2 y}{1+x^3}$ satisfying the initial conditions $x=1, y=2$. (15%)

2. Solve the differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=u(x), y(0)=y'(0)=0$ using Laplace transform, where the input function $u(x)$ is the unit step function as below. Explain whether the problem is stable or not. (17%)



3. Find the solution $y(x)$ of the two-point boundary value problem $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4y=3-2x, y(0)=\frac{11}{8}$ and $y'(1)=\frac{1}{2}$. Write the homogenous solution, particular solution and the overall solution satisfying the conditions. (18%)

4. Find the eigenvalues and a basis of eigenvectors for the following matrix. (10%)

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

5. Let x, y, z be right-handed Cartesian coordinate, and let $\bar{v}(x,y,z)=v_1\bar{i}+v_2\bar{j}+v_3\bar{k}$ be a differentiable vector function. What is the definition of the curl of the vector function $\bar{v}$? (5%) Prove Curl(gradient f) = 0, if f is a twice continuously differentiable scalar function. (5%)

6. The two-dimensional heat equation is

$$\frac{\partial u}{\partial t}=c^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right).$$

If the heat flow is steady, then $\partial u/\partial t=0$, and the heat equation reduces to Laplace's equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0.$$

The boundary conditions are shown in the following figure. Solve the differential equation. (30%)