1. Find an integrating factor and solve $\frac{dy}{dx}+\frac{x^2+y^2+x}{xy}=0$

   Given $y=1$ at $x=1$, what is y at $x=0.5$? (15%)

2. Find the solution $y(t)$ of the two-point boundary value problem

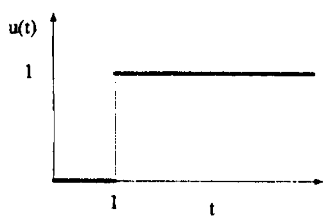
   $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+4y=4t^2+e^t$, $y(0)=0$, $y'(1)=0$ (15%)

3. For a system described by the following equations

   $\frac{dy}{dt}-x=0,\ y(0)=0$

   $\frac{dx}{dt}+2x+2y=4u(t-1), x(0)=0$

   where u(t) is the unit step function as below:



   (a) Find the Laplace transform Y(s) of y(t). Be sure to eliminate X(s) from the expression.

   (b) Explain that the system is stable or unstable.

   (c) Determine the final value of y(t) as $t\rightarrow\infty$.

   (d) Find the solution $x(t)$ and $y(t)$. (20%)

4. Transform the equation $9x_1{}^2-6x_1x_2+x_2{}^2=160$ to the form of $ay_1{}^2+by_2{}^2=160$, where a and b are constants. y_1 and y_2 are the variables of linear combinations of x_1 and x_2. (10%)

5. Calculate the length of the circular helix: $\vec{r}(t) = a\cos(t)\,\hat{i} + a\sin(t)\,\hat{j} + t\,\hat{k}$ from $(a, 0, 0)$ to $(a, 0, \pi)$. (10%)

6. Prove the divergence theorem of Gauss $\iiint_T \nabla \bullet \vec{F}\, dV = \iint_S \vec{F} \bullet \vec{n}\, dA$, where T is a closed bounded region in space whose boundary is a piecewise smooth orientable surface S. $\vec{n}$ is the outer unit normal vector of S. $\vec{F}$ is a vector function that is continuous and has continuous first partial derivatives in some domain containing T.

(10%)

7. Suppose heat is not lost from the lateral surface of a thin rod of length L into a surrounding medium. Then, the heat equation takes on the form

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2},\ 0 < x < L,\ t > 0, \text{ where } c^2 \text{ is a constant.}$$

Find the temperature $u(x, t)$ if the initial temperature is $f(x)$ throughout and the ends $x = 0$ and $x = L$ are insulated.

(Hint: insulation means temperature gradient is zero) (20%)