

國立中正大學九十二學年度碩士班招生考試試題

系所別：化學工程學系

科目：工程數學

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1. (10 points) Find the solution of the ordinary differential equation

$$y'' + 5y' + 4y = 3 - 2x, \quad y(0) = \frac{11}{8}, \quad y'(0) = \frac{1}{2}$$

2. (10 points) Solve the first order differential equation

$$\frac{dy}{dx} = \frac{-4x + y - 2}{x + y + 3}$$

3. (10 points) For a system described by the following equations

$$\frac{dx}{dt} + x + y = e^{-2t}, \quad x(0) = 1$$

$$\frac{dy}{dt} + 4x + y = 0, \quad y(0) = 0$$

(a) Find the Laplace transform $X(s)$ of $x(t)$. Be sure to eliminate

$Y(s)$ from this expression. (b) Find the solution $y(t)$.

4. (10 points) A square matrix \mathbf{A} is called orthogonal if $\mathbf{A}^T = \mathbf{A}^{-1}$. (a) Prove that the determinant of an orthogonal matrix must be 1 or -1. (b) Prove that the inverse \mathbf{A}^{-1} has the eigenvalues $1/\lambda_1, \dots, 1/\lambda_n$, where $\lambda_1, \dots, \lambda_n$ are the eigenvalues of the matrix \mathbf{A} .

5. (10 points) Determine λ such that the points $\mathbf{a} = i + j + k$, $\mathbf{b} = 2i - 4k$ and $\mathbf{c} = i + \lambda j + 3k$ are coplanar.

6. The Fourier integral of $f(x)$ is

$$f(x) = \int [a(w)\cos(wx) + b(w)\sin(wx)]dw.$$

- (a) (5 points) What are general expressions for $a(w)$ and $b(w)$ if $f(x)$ is an even function?

(b) Set $\exp(-x^2) = \int [F(w)\cos(wx)]dw.$

- ① (5 points) Evaluate $F(0)$.
② (10 points) Derive the explicit expression for $F(w)$.

7. ① (5 points) Write down the generalized Cauchy integral formula.

- ② (5 points) Evaluate

$$\oint \frac{e^{2z}}{(z+1)^2} dz.$$

8. (20 points) Solve the partial differential equation given below.

$$\frac{\partial U}{\partial t} = \alpha \frac{\partial^2 U}{\partial y^2}.$$

The associated boundary conditions and initial condition are

$$y = 0, \quad U = U_0$$

$$y \rightarrow \infty, \quad U = 0,$$

$$t \leq 0, \quad U = 0$$