

國立中正大學九十學年度碩士班招生考試試題

系所別：化學工程學系

科目：工程數學

共3頁, 第1頁

1. The Laplace transform of a function $f(t)$ is defined by

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

- (a) Write down the conditions that $f(t)$ should meet so that the Laplace integral (the integral on the left-hand side of the above equation) exists. (4 points)
- (b) Let $f(t) = te^{-t}$. What is the region of s that ensures the existence of the Laplace integral. (3 points)
- (c) Find the Laplace transform of the function: (3 points)

$$g(t) = \begin{cases} 1, & \text{for } 0 < t \leq 1 \\ 3, & \text{for } 1 < t \leq 2 \\ 2, & \text{for } 2 < t \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

- (d) Find the inverse Laplace transform of $F(s) = (e^{-s} - e^{-2s})/s$, and denote it by $f(t)$. (5 points)
- (e) Find the convolution integral of $f(t)$ and $g(t)$, and denote it by $h(t)$. Plot $h(t)$ for $0 \leq t \leq 5$ and find the Laplace transform of $h(t)$. (Note: $f(t)$ and $g(t)$ are given in (c) and (d), respectively) (5 points)
2. A single, isothermal, well-mixed, constant-volume CSTR is considered in this question. The chemical reaction is



which is first-order with the forward and reverse rate constants k_1 and k_2 , respectively. Only component A appears in the feed. The system is initially at steady state and experiences a step in the concentration of A in the feed. Formulate a model to describe this system, and solve for the concentrations in the reactor. (10 points)

共3頁

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3. Given the system of equations:

$$-x_1 + x_2 + 2x_3 = 2$$

$$3x_1 - x_2 + x_3 = 6$$

$$-x_1 + 3x_2 + 4x_3 = 4$$

- (a) How to determine if the system has a unique solution. (5 points)
 - (b) Find the solution step by step with Gauss elimination method. (5 points)
4. Suppose that the function $f(a, b, x, y)$ satisfies the relation

$$f(a, b, X, Y) = k^h f(a, b, x, y) \quad (A)$$

where

$$X = kx \text{ and } Y = ky$$

- (a) Find the total differentials of both sides of (A). (5 points)
 - (b) Using the relation $dX = kdx + xdk$, $dY = kdy + ydk$ and the result of (a) to show that (5 points)
- $$x \frac{\partial}{\partial x} f(a, b, x, y) + y \frac{\partial}{\partial y} f(a, b, x, y) = hf(a, b, x, y)$$
5. (a) Give the parametric representation $\mathbf{r}(u, v)$ of a sphere: $x^2 + y^2 + z^2 = a^2$. (5 points)
- (b) Use the above result to obtain an expression for the outwardly directed unit normal vector at point $P(x, y, z)$ on the sphere. (5 points)
 - (c) Use the above results to evaluate the surface area of the sphere. (10 points)

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6. The steady-state heat conduction problem in a solid sphere leads to the Laplacian:

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) = 0.$$

Find the temperature distribution within the sphere $T(r, \theta)$ if the temperature on the outer surface $r = R$ is maintained at $T = f(\theta)$. (Hint: you may find it useful to make the change of variable: $w = \cos \theta$ in setting up the equation for the θ component) (20 points)

7. Evaluate the following integration:

$$\oint_C \frac{z+1}{z(z-2)(z-4)^2} dz,$$

where z is a complex variable, and C is the counterclockwise unit circle $|z-3|=2$. (10 points)