國立中正大學八十九學年度碩士班招生考試試題

系所別：化學工程學系　　　　科　目：工程數學

1. Consider the stirred-tank reactor shown in Fig. 1. The reaction occurring is

$$A \rightarrow B$$

and it proceeds at a rate

$$r = kC_o(t)$$

where

$$
\begin{aligned}
r &= \text{moles A reacting/(volume time)} \\
k &= \text{reaction velocity constant} \\
C_o(t) &= \text{concentration of A in reactor, moles/volume} \\
V &= \text{volume of mixture in reactor} \\
F &= \text{constant feed rate, volume/time} \\
C_i(t) &= \text{concentration of A in feed stream, moles/volume}
\end{aligned}
$$

Assume constant density and constant V, derive the differential equation relating the concentration in the reactor to the feed-stream. Sketch the response of the reactor to a unit-step change in C_i. (14 points)

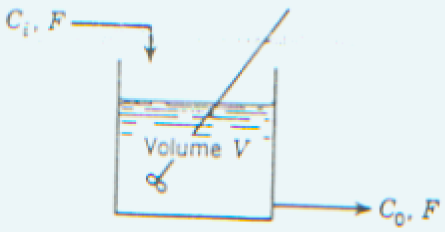

Fig. 1

2. The dynamics of a linear system is described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{du(t)}{dt} + 3u(t),\ t \geq 0$$

where $u(t)$ and $y(t)$ denote the input and output variables of the system, respectively.

(a) If the input variable is given by $u(t) = e^{-3t}$, find $y(t)$ in terms of $y(0)$ and $y'(0) = dy(0)/dt$. (6 points)

(b) Find $y(0)$ and $y'(0)$ such that $y(t) = 0$ for $t > 0$. (6 points)

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3. Let

$$A = \begin{bmatrix} -1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

   (a) Find the eigenvalues and the corresponding eigenvectors of A. (6 points)

   (b) Find a matrix P such that PAP^{-1} is a diagonal matrix. (6 points)

4. (a) Describe a procedure of finding all the real roots of a polynomial $p(x)$ in the interval $[\underline{x}, \overline{x}]$. (6 points)

   (b) How to determine if three twice differentiable functions $y_1(x), y_2(x), y_3(x)$ are linear independent. (6 points)

5. Solve the diffusion problem in a circular membrance:

$$\frac{\partial c}{\partial t} = \frac{\partial^2 c}{\partial r^2} + \frac{1}{r}\frac{\partial c}{\partial r},$$

   subject to the three conditions: (20 points)

$$\begin{aligned} c(R,t) &= 0, \qquad t > 0 \\ c(r,0) &= f(r), \quad 0 < r < R, \\ \lim_{t\to\infty} c(r,t) &= 0, \quad 0 \le r \le R. \end{aligned}$$

6. Evaluate the following flux integral:

$$\int_S\int \boldsymbol{F}\cdot\boldsymbol{n}\,dA,$$

   where $\boldsymbol{F} = x^3\vec{i} + x^2y\,\vec{j} + x^2z\,\vec{k}$, and S is the closed surface: $x^2 + y^2 = a^2, \quad 0 \le z \le b$. (15 points)

7. (a) Evaluate i^i, where i is the imaginary unit. (5 points)

   (b) Evaluate the following integral: (10 points)

$$\int_0^\infty \frac{1}{1+x^4}\,dx.$$