

國立中正大學八十九學年度碩士班招生考試試題

系所別：化學工程學系

科目：工程數學

1. Consider the stirred-tank reactor shown in Fig. 1. The reaction occurring is



and it proceeds at a rate

$$r = kC_a(t)$$

where

- r = moles A reacting/(volume time)
- k = reaction velocity constant
- $C_a(t)$ = concentration of A in reactor, moles/volume
- V = volume of mixture in reactor
- F = constant feed rate, volume/time
- $C_i(t)$ = concentration of A in feed stream, moles/volume

Assume constant density and constant V , derive the differential equation relating the concentration in the reactor to the feed-stream. Sketch the response of the reactor to a unit-step change in C_i . (14 points)



Fig. 1

2. The dynamics of a linear system is described by the differential equation

$$\frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2y(t) = \frac{du(t)}{dt} + 3u(t), t \geq 0$$

where $u(t)$ and $y(t)$ denote the input and output variables of the system, respectively.

- (a) If the input variable is given by $u(t) = e^{-3t}$, find $y(t)$ in terms of $y(0)$ and $y'(0) = dy(0)/dt$. (6 points)
- (b) Find $y(0)$ and $y'(0)$ such that $y(t) = 0$ for $t > 0$. (6 points)

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3. Let

$$A = \begin{bmatrix} -1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

- (a) Find the eigenvalues and the corresponding eigenvectors of A . (6 points)
- (b) Find a matrix P such that PAP^{-1} is a diagonal matrix. (6 points)
4. (a) Describe a procedure of finding all the real roots of a polynomial $p(x)$ in the interval $[\underline{x}, \overline{x}]$. (6 points)
- (b) How to determine if three twice differentiable functions $y_1(x), y_2(x), y_3(x)$ are linear independent. (6 points)
5. Solve the diffusion problem in a circular membrane:

$$\frac{\partial c}{\partial t} = \frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r}$$

subject to the three conditions: (20 points)

$$\begin{aligned} c(R, t) &= 0, & t > 0 \\ c(r, 0) &= f(r), & 0 < r < R, \\ \lim_{t \rightarrow \infty} c(r, t) &= 0, & 0 \leq r \leq R. \end{aligned}$$

6. Evaluate the following flux integral:

$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dA,$$

where $\mathbf{F} = x^3 \mathbf{i} + x^2 y \mathbf{j} + x^2 z \mathbf{k}$, and S is the closed surface: $x^2 + y^2 = a^2$, $0 \leq z \leq b$. (15 points)

7. (a) Evaluate i^i , where i is the imaginary unit. (5 points)
- (b) Evaluate the following integral: (10 points)

$$\int_0^{\infty} \frac{1}{1+x^2} dx.$$