國立中正大學八十八學年度碩士班招生考試試題

系所別：化學工程學系　　　　科　目：工 程 數 學

1. Solve the following initial value problems: (15 points for each)

(i) $y'' - y = e^x(1 + \sin x)$　　$y(0) = \frac{8}{5}$,　$y'(0) = -\frac{1}{10}$

(ii) $y'' - 5y' + 6y = r(t)$, $r(t) = 4e^t$ if $0 < t < 2$ and 0 if $t > 2$;
$y(0) = 1$, $y'(0) = -2$.

2. (i) For a set of eigenfunctions, what two properties it must have to be practically useful? (5 points)

(ii) Give the solution of $y'' + k^2xy = 0$ (Try $y = u\sqrt{x}$, $\frac{2}{3}kx^{3/2} = z$). (10 points)

3. Evaluate the surface integral

$$\int\int_S \boldsymbol{n} \cdot \boldsymbol{F}\, dA \quad (\boldsymbol{n} \text{ is the ourwardly directed unit normal vector of } dA)$$

for $\boldsymbol{F} = (x + z)\vec{i} + (y + z)\vec{j} + (x + y)\vec{k}$,　$S : x^2 + y^2 + z^2 = 4$, $z \geq 0$. (15 points).

4. The temperature distribution $T(x,t)$ in a slab is governed by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

where α is constant. The slab, $0 \le x \le L$, is initially at a constant temperature T_0. For time $t > 0$ the boundaries at $x = 0$ and $x = L$ are kept at zero temperature. Obtain an expression for the temperature distribution in the slab for times $t > 0$. (20 points)

5. The matrix $\mathbf{A} = \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix}$. Find the eigen values and the corresponding eigen vectors of $\mathbf{A}$. Evaluate $\mathbf{A}^{10}$. (10 points)

6. Find a solution (4 digits accuracy) of the equation $f(x) = x^3 + x - 1 = 0$ by Newton's method, starting from the given point $x_0 = 1$. You must write out your calculated procedures. (Hint: Newton's method: $x_{n+1} = x_n - f(x_n)/f'(x_n)$). (10 points)