

國立中正大學八十六學年度碩士班考試試題

所 別：化學工程研究所

科 目：工程數學

1. Solve the following equation

$$\frac{d^2x}{dt^2} + 4x = 3e^{2t} + 7\cos(t). \quad (10分)$$

2. (a) What are the similarities and differences between Fourier and Laplace Transforms? (8分)

- (b) Solve the following equation by Laplace transform

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = e^{5t}, \quad (10分)$$

$$x = 1 \text{ and } \frac{dx}{dt} = 3 \text{ at } t = 0$$

3. Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & -2 \\ -5 & 3 & 8 \end{bmatrix} \quad (10分)$$

4. (a) For a scalar field $c(x,y,z)$, show that $\text{div}(\text{grad } c) = 0$. (8分)

- (b) Let x and y be defined as functions of u and v by the implicit equations

$$x^2 + y^2 + u^2 + v^2 = 1$$

$$x^2 + 2y^2 - u^2 + v^2 = 1 \quad (8分)$$

express $\frac{\partial x}{\partial u}$ and $\frac{\partial^2 y}{\partial u^2}$ in terms of u and v .

5. Show that the set $s = \{\cos(nx), n = 1, 2, \dots\}$, is an orthogonal set on the interval $[0, \pi]$. If the interval is changed to $[0, 1]$, modify s to define a new orthogonal set. (14分)

6. (a) Show that the second derivative $\frac{d^2x}{dt^2}$ at $t = t_i$ can be approximated by the finite difference

$$\frac{x(t_i + \Delta t) - 2x(t_i) + x(t_i - \Delta t)}{\Delta t^2} \quad (8分)$$

- (b) Choosing $\Delta t = 0.5$, use the finite difference method to solve $x(1.5)$, $x(2)$ and $x(2.5)$ for

$$\frac{d^2x}{dt^2} - (1 - \frac{t}{5})x = t \text{ with } x(1) = 2 \text{ and } x(3) = -1. \quad (8分)$$

7. Solve the PDE $\frac{\partial u}{\partial t} = \frac{d^2u}{dx^2}$

$$\text{with boundary conditions: } u(0,t) = 0, \frac{\partial u(1,t)}{\partial x} + u(1,t) = 0, \quad (16分)$$

and initial condition: $u(x,0) = x$.

How does solution $u(x,t)$ behave for large time t ?