

國立中正大學八十五學年度碩士班考試試題

所 別：化學工程研究所

科 目：工程數學

(可攜帶電子計算器)

1. Consider the PDE

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} - u(x,t), \quad 0 \leq x \leq 1, \quad 0 < t < \infty \quad (1)$$

with the following boundary and initial conditions:

$$u(0,t) = u(1,t) = 0$$

$$u(x,0) = \sin(\pi x) + 0.5 \sin(3\pi x)$$

Equation (1) can be transformed into a simpler PDE by letting

$$u(x,t) = \exp(-t) w(x,t)$$

(a) Derive the transformed PDE as well as the boundary and initial conditions. (5分)

(b) Find the solution $u(x,t)$. (10分)

2. The Simpson's formula for numerical integration of a function $f(x)$ is given by

$$\begin{aligned} \int_{x_1}^{x_1+2\Delta x} f(x) dx &= \int_{x_1}^{x_1+\Delta x} f(x) dx + \int_{x_1+\Delta x}^{x_1+2\Delta x} f(x) dx \\ &\approx \frac{\Delta x}{3} (f(x_1) + 4f(x_1+\Delta x) + f(x_1+2\Delta x)) \end{aligned}$$

(a) Show how to reach the above formula. (8分)

(b) Compute $\int_0^1 x^2 dx$ by the Simpson's formula with $\Delta x = \frac{1}{4}$, and compare the result with the exact integration value. (5分)

3. Evaluate the following integrals

(a) $\int_0^\pi \int_0^a r^2 \sin^2(\theta) r dr d\theta$ (5分)

(b) $\int_{-\infty}^{\infty} \frac{x \cos(x)}{x^2 + 1} dx$ (5分)

4. State briefly the meanings of the following terms and their connections to Chemical Engineering. (12分)

(a) Divergence of a vector field.

(b) Curl of a vector field.

(c) Gradient of a scalar field.

國立中正大學八十五學年度碩士班考試試題

所別：化學工程研究所

科目：工程數學

(可攜帶電子計算器)

5. (a) Find the solution of the ordinary differential equation

$$y'' + 5y' + 4y = 3 - 2x, \quad y(0) = \frac{11}{8}, \quad y'(0) = \frac{1}{2} \quad (10\text{分})$$

- (b) Find the eigenvalues and eigenfunctions of the problem

$$(xy')' + \lambda x^{-1}y = 0, \quad y(1) = 0, \quad y'(e) = 0 \quad (10\text{分})$$

6. Find the Laplace transform $Y(s)$ and the final value, $y(t \rightarrow \infty)$, of the following problem.

$$x(t) = 2z(t) + 0.5 \frac{dz}{dt}$$

$$3 \frac{dy}{dt} + y(t) = 0.5x(t - 0.5)$$

$$z(t) = r(t) - y(t)$$

where all initial conditions of the problem are assumed to be zero, and

$$r(t) = \begin{cases} 0, & \text{for } t < 0 \\ 1, & \text{for } t \geq 0 \end{cases} \quad (10\text{分})$$

7. Find the characteristic equation of the matrix $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -d & -c & -b & -a \end{bmatrix}$

Find the eigenvalues of this matrix if $a = 4$, $b = 6$, $c = 4$ and $d = 1$. (10分)

8. A farmer fills his silo with chopped corn. The entire corn plant (leaves, stem, and ear) is cut up into small pieces and blow into the top of the cylindrical silo at a rate W_0 . The process is sketched in Figure 1. The diameter of the silo is D and its height is H . The density of the chopped corn in the silo varies with the depth of the bed. The density ρ at a point that has z feet of material above it is

$$\rho(z) = \rho_0 + \beta z$$

where ρ_0 and β are constants.

- (a) Write the equations that describe the system and show how the height of the bed $h(t)$ varies as a function of time.

- (b) What is the total weight of corn fodder that can be stored in the silo? (10分)

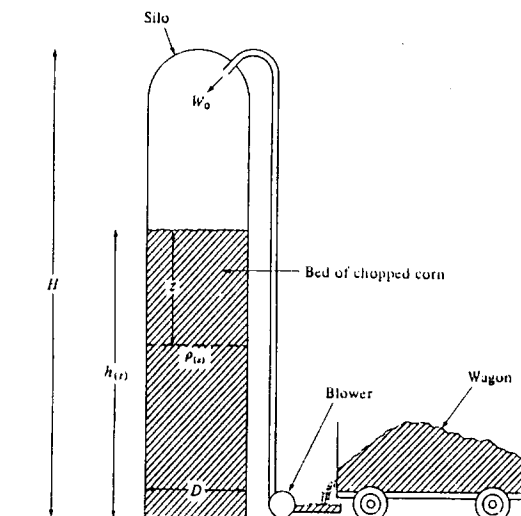


Figure 1.