國立中正大學八十二學年度碩士班考試試題

所　別：化學工程研究所　　　　科　目：工程數學

1. Solve the initial value problem

$$\frac{dy_1}{dt} = -4y_1 + 3y_2$$

$$\frac{dy_2}{dt} = 2y_1 - 3y_2$$

where the initial values are $y_1(t_0) = y_2(t_0) = 1$. （15分）

2. Solve the first order differential equation

$$\frac{dy}{dx} = \frac{-4x + y - 2}{x + y + 3}.$$ （10分）

3. Determine λ such that the points $\mathbf{a} = \mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 4\mathbf{k}$, and $\mathbf{c} = \mathbf{i} + \lambda\mathbf{j} + 3\mathbf{k}$ are coplanar. （7分）

4. A square matrix $\mathbf{A}$ is called orthogonal if $\mathbf{A}^T = \mathbf{A}^{-1}$. (a) Prove that the determinant of an orthogonal matrix must be 1 or -1. (b) Prove that the inverse $\mathbf{A}^{-1}$ has the eigenvalues $1/\lambda_1, \cdots, 1/\lambda_n$, where $\lambda_1, \cdots, \lambda_n$ are the eigenvalues of the matrix $\mathbf{A}$. （8分）

5. Evaluate the integral

$$\int_{(0,1,2)}^{(2,\pi,0)} [\cos(xy)(y\,dx + x\,dy) + dz].$$ （10分）

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6. (a) Let $\{\phi_n(x) : n = 1, 2, 3, ...\}$ be a closed orthonormal set over some interval $x \in (a, b)$. Show that if $a_1\phi_1(x) + a_2\phi_2(x) + ...$ is the expansion of a function f(x), then

$$\sum_{i=1}^{\infty} a_i^{\,2} = \int_a^b [f(x)]^2\, dx. \quad (5分)$$

   (b) Explain the following terms: Taylor series, Fourier series, Laurent series. (5分)

7. (a) Write down the formulas for the Fourier and Laplace transforms of a function f(t). (5分)

   (b) What are the differences between the Fourier and Laplace transforms ? (5分)

8. (a) How to determine if the second-order differentiable functions $f_1(t), f_2(t)$, and $f_3(t)$ are linearly independent ? (5分)

   (b) How to use the Routh-Hurwitz stability test to determine if all the roots of a fourth-order polynomial $p(s) = a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4$ have negative real part ? (5分)

9. (a) Evaluate $\int_C \frac{dz}{z}$ around a circle with its center at the origin. (5分)

   (b) Find the Laplace transform of the nth-degree Laguerre polynomial $L_n(t)$ defined by

$$L_n(t) = \frac{e^x}{n!}\frac{d^n(x^n e^{-x})}{dx^n}. \quad (5分)$$

10. Find the solution T(x,t) of the following partial differential equation for $t > 0$ and $0 < x < L$:

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2}, \qquad T = -(T_1 - T_0)x/L \quad for \quad t = 0$$

$$T = 0 \quad for \quad x = 0$$

$$T = 0 \quad for \quad x = L \qquad (10分)$$