

國立中正大學八十二學年度碩士班考試試題

所 別：化學工程研究所

科 目：工程數學

1. Solve the initial value problem

$$\frac{dy_1}{dt} = -4y_1 + 3y_2$$

$$\frac{dy_2}{dt} = 2y_1 - 3y_2$$

where the initial values are $y_1(t_0) = y_2(t_0) = 1$.

(15分)

2. Solve the first order differential equation

$$\frac{dy}{dx} = \frac{-4x + y - 2}{x + y + 3}$$

(10分)

3. Determine λ such that the points $\mathbf{a} = \mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 4\mathbf{k}$, and $\mathbf{c} = \mathbf{i} + \lambda\mathbf{j} + 3\mathbf{k}$ are coplanar.

(7分)

4. A square matrix \mathbf{A} is called orthogonal if $\mathbf{A}^T = \mathbf{A}^{-1}$. (a) Prove that the determinant of an orthogonal matrix must be 1 or -1. (b) Prove that the inverse \mathbf{A}^{-1} has the eigenvalues $1/\lambda_1, \dots, 1/\lambda_n$, where $\lambda_1, \dots, \lambda_n$ are the eigenvalues of the matrix \mathbf{A} .

(8分)

5. Evaluate the integral

$$\int_{(0,1,2)}^{(2,\pi,0)} [\cos(xy)(ydx + xdy) + dz]$$

(10分)

(續下頁)

國立中正大學八十二學年度碩士班考試試題

所 別：化學工程研究所

科 目：工程數學

6. (a) Let $\{\phi_n(x) : n = 1, 2, 3, \dots\}$ be a closed orthonormal set over some interval $x \in (a, b)$. Show that if $a_1\phi_1(x) + a_2\phi_2(x) + \dots$ is the expansion of a function $f(x)$, then

$$\sum_{i=1}^{\infty} a_i^2 = \int_a^b [f(x)]^2 dx. \quad (5分)$$

- (b) Explain the following terms: Taylor series, Fourier series, Laurent series. (5分)
7. (a) Write down the formulas for the Fourier and Laplace transforms of a function $f(t)$. (5分)
- (b) What are the differences between the Fourier and Laplace transforms? (5分)
8. (a) How to determine if the second-order differentiable functions $f_1(t)$, $f_2(t)$, and $f_3(t)$ are linearly independent? (5分)
- (b) How to use the Routh-Hurwitz stability test to determine if all the roots of a fourth-order polynomial $p(s) = a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4$ have negative real part? (5分)
9. (a) Evaluate $\int_C \frac{dz}{z}$ around a circle with its center at the origin. (5分)
- (b) Find the Laplace transform of the n th-degree Laguerre polynomial $L_n(t)$ defined by

$$L_n(t) = \frac{e^x}{n!} \frac{d^n(x^n e^{-x})}{dx^n}. \quad (5分)$$

10. Find the solution $T(x,t)$ of the following partial differential equation for $t > 0$ and $0 < x < L$:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, \quad T = -(T_1 - T_0)x/L \quad \text{for } t = 0$$

$$T = 0 \quad \text{for } x = 0$$

$$T = 0 \quad \text{for } x = L$$

(10分)