

國立中正大學101學年度學士班二年級轉學生招生考試試題
數學系、地球與環境科學系、物理學系
學系別： 資訊工程學系、電機工程學系、機械工程學系 科目：微積分
化學工程學系

第 1 節

第 1 頁，共 2 頁

一、填充題(每個空格7分, 共56分)

1. If $f(x) = \exp(g(x))$, where $g(x) = \int_0^{\sin(\pi x)} \sqrt{1+t^2} dt$, find $f'(1)$. (a)

2. Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-3)^n}{n^2-1}$. (b)

3. For what value of C does the equation $\ln x = Cx^3$ have exactly one solution? (c)

4. Find the linearization $L(x, y) = Ax + By + C$ of the function $f(x, y) = x\sqrt{y}$ at the point $(-5, 4)$. (d)

5. Find the arc length of the curve $y = \frac{x^2}{2}$, $x \in [0, 1]$. (e)

6. The base of a solid is the ellipse $x^2 + 4y^2 = 4$, and every parallel cross sections perpendicular to the x -axis are equilateral triangles (等邊三角形). Find the volume of the solid. (f)

7. Find the area of the region enclosed by the curve $r^2 = 4 \sin 2\theta$. (g)

8. Let S be the surface of the solid E that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$ and $\mathbf{F}(x, y, z) = (z, y, \sin(x+y))$. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S} =$
(h)

國立中正大學101學年度學士班二年級轉學生招生考試試題

數學系、地球與環境科學系、物理學系

學系別：資訊工程學系、電機工程學系、機械工程學系
化學工程學系

科目：微積分

第 1 節

第 2 頁，共 2 頁

二、計算題(44分，這部分必須有完整計算過程，僅有答案而沒有計算過程得0分)

1. (4+4+4=12分) Sketch the solid whose volume is given by $\int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} f(x, y, z) dy dx dz$.
Then, rewrite this iterated integral in the following orders:

(a) $\iiint f(x, y, z) dx dz dy$ (b) $\iiint f(x, y, z) dy dz dx$

2. (5+5=10分) Please complete the following steps to compute the improper integral $\int_0^{\infty} e^{-x^2} dx$:

(a) First of all, compute the double integral $\iint_{R^2} e^{-x^2-y^2} dA$

- (b) Apply Fubini's theorem and symmetric property of the function e^{-x^2} to find the integral $\int_0^{\infty} e^{-x^2} dx$.

3. (4+2+6=12分) (a) Derive the formula: $\int_0^1 x^n (\ln x)^k dx = \frac{-k}{n+1} \int_0^1 x^n (\ln x)^{k-1} dx$

(b) Derive the formula $\int_0^1 x^n (\ln x)^n dx = \frac{(-1)^n n!}{(n+1)^{n+1}}$

- (c) Use the formula $x^x = e^{x \ln x}$ and the Maclaurin series for e^x to derive the formula

$$\int_0^1 x^x dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^n}$$

4. (10分) Let $f(x, y) = 8xy - 2x - 4y + 5$. Find the absolute minimum value of the function $f(x, y)$ on the set D , where D is the region bounded by the parabola $y = x^2$ and the line $y = 4$.