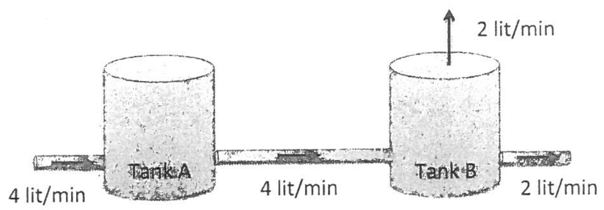
1. Consider the matrix

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 12 & 2 & 0 \\ 21 & -6 & 9 \end{bmatrix}$$

(a) Show that the eigenvectors are linearly independent. (10%)

(b) Diagonalize the matrix A. (10%)

2. Consider two Tanks, Tank A and Tank B as in the figure below. Tank A and Tank B initially contains 100 liter of pure water. Water containing salt at a concentration of 30 g/liter flows into Tank A at a rate of 4 lit/min. Mixture flows from Tank A to Tank B at 4 lit/min. Water evaporates from Tank B at 2 lit/min and the mixture of Tank B flows out at 2 lit/min. The mixtures are kept uniform by stirring in both tanks. Please use ***Laplace transform*** to find the amounts of salt $y_A(t)$ and $y_B(t)$ in Tank A and Tank B, respectively, where t is time. (20%)



3. Please solve the initial value problem. (20%)

$$y'' + 2y' + 0.75y = 2\cos x - 0.25\sin x + 0.09x$$

$$y(0) = 2.78, \qquad y'(0) = -0.43$$

4. (a) Find the Fourier series representation for the following function. (10%)

$$f(x) = x, \qquad -\pi < x \leq \pi$$

(b) Show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$ (5%)

(c) Find the Fourier series representation for the following function. (5%)

$$f(x) = x, \qquad -2 < x \leq 2$$

5. Please solve the partial differential equation using ***separation of variables***. (20%)

$$\frac{\partial T}{\partial t} = \alpha^2 \frac{\partial^2 T}{\partial x^2}$$

B.C. $T(0,t) = 0, \quad T(L,t) = 0, \quad$ *for all t,*

I.C. $T(x,0) = 100\sin\left(\frac{\pi x}{L}\right)$

Table of Laplace Transforms

$f(t)$	$L(f)(t)$
1	$\frac{1}{s}, s>0$
t^n	$\frac{n!}{s^{n+1}}, s>0$
$\sin(at)$	$\frac{a}{s^2+a^2}, s>0$
$\cos(at)$	$\frac{s}{s^2+a^2}, s>0$
e^{at}	$\frac{1}{s-a}, s>a$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}, s>a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}, s>a$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}, s>a$