1. **(25%)** Classify the following differential equations: state their order, degree and linearity, and indicate the independent variable(s) and dependent variable(s) of the equations. DO NOT attempt to solve them.

(a) $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=10sinx$

(b) $\frac{dQ}{dt}=e^Q-1$

(c) $cosx\left(\frac{dy}{dx}\right)^6+sinx\left(\frac{d^2y}{dx^2}\right)^3=0$

(d) $(\frac{d^5x}{dt^5})^2=x$

(e) $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$

2. **(25%)** A spring-mass-dashpot system. 2nd order homogeneous linear ordinary differential equation (ODE) is defined as: $\mathbf{y''+ay'+by=0}$, where coefficients ***a** and **b*** are constant.

(a) Find the characteristic equation of the system, (6%)

(b) If the roots (r_1 and r_2) of the characteristic equation are real and unequal ($r_1 \neq r_2$), find the general solution, (6%)

(c) If the coefficients a= 4 and b= 3, $y(0)$ =1, and $y'(0)$ = 0, find the particular solution (7%), and

(d) draw a curve (y v.s. t, $0 < t < 2$) roughly representing the particular solution. (6%)

(hint : try a solution of the function, $y=e^{rt}$, where y: displacement, t: time, and r: root in the case of a spring-mass-dashpot system)

3. **(15%)** Using Laplace transform to solve : $\mathbf{y''-y=t}$, $\mathbf{y(0)=1}$, $\mathbf{y'(0)=1}$

4. **(15%)** (a) Expand $\mathbf{f(x)=x^2}$, $0<x<2\pi$ in a Fourier series if the period is 2π , (8%)

(b) find $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots=?$ (7%)

5. **(20 %)** Solve the partial differential equation, $\frac{\partial T}{\partial t}=\frac{\partial^2T}{\partial x^2}$, by the method of separating variables.

Initial condition: $\mathbf{T(x,0)=f(x)}$

Boundary conditions:

(1) $\frac{\partial T(0,t)}{\partial x}=\mathbf{0}$

(2) $\frac{\partial T(l,t)}{\partial x}=-\mathbf{T}(\boldsymbol{l},\mathbf{t})$