1. The half-life of ^{14}C is 5715 years, and its radioactive decay is governed by the ODE $y' = ky$. What should be the ^{14}C content (in percent of initial content y_0) of a fossilized tree that is claimed to be 3000 years old? (20%)

2. Please solve the following initial value problem with the help of matrix. (20%)
   $y_1' = 2y_1 + 5y_2$
   $y_2' = 5y_1 + 12.5y_2$
   $y_1(0) = 12, \quad y_2(0) = 1$

3. Please solve the following initial value problem by using the Laplace transform. (20%)
   $y_1' + 6y_1 - 4y_2 = 0$
   $y_2' + 4y_1 - 4y_2 = 0$
   $y_1(0) = -2, \quad y_2(0) = -7$

4. Please find the eigenvalues and eigenfunctions of the following boundary value problem, and show the orthogonality of eigenfunctions. (20%)
   $y'' + \lambda y = 0, \quad y(0) = 0, \quad y'(L) = 0$

5. Please find a basis of eigenvectors and diagonalize the following matrix. (20%)

$$\begin{bmatrix} -1 & 2 & -2 \\ 2 & 4 & 1 \\ 2 & 1 & 4 \end{bmatrix}$$