

## 臺灣綜合大學系統 105 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 A	類組代碼	E00
		科目碼	E0011
※本項考試依簡章規定各考科均「不可以」使用計算機		本試題共計	1 頁

※請於答案卷作答，題號並請標示清楚！

填充題：每格 5 分，8 格總共 40 分

1.  $\lim_{x \rightarrow 0} \frac{x^2 + \sin^2 x}{1 - \cos x} = \underline{\quad 1(a) \quad}$ ;  $\lim_{x \rightarrow 0^+} (e^x + x)^{\frac{1}{x}} = \underline{\quad 1(b) \quad}$

2. Let  $x^2 + xy + 2y^2 = 1$ .

Find  $\frac{dy}{dx} = \underline{\quad 2(a) \quad}$  and  $\frac{d^2y}{dx^2} = \underline{\quad 2(b) \quad}$  at  $(x, y) = (1, 0)$ .

3. Find the interval =  $\underline{\quad 3(a) \quad}$  of convergence of the power series

$\sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{2n+1}$ , and its sum =  $\underline{\quad 3(b) \quad}$

4. The graph of the equation  $y = \frac{x}{(x+3)^2}$  is strictly increasing on the interval

=  $\underline{\quad 4(a) \quad}$  and concave upward on the interval =  $\underline{\quad 4(b) \quad}$ .

計算與證明題：每題 10 分，6 題總共 60 分

5. Evaluate  $\int_0^{\infty} x^2 e^{-x^2} dx$  from the known integral  $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$

6. Show that  $F(x) = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin\left(\frac{x}{a}\right)$ ,  $a > 0$  is an anti-derivative for  $f(x) = \sqrt{a^2 - x^2}$ .

7. Let  $z = \ln\left(\frac{1+x}{1+y}\right)$  where  $x = \cos t$ ,  $y = \tan t$ . Use the chain rule to find the value of  $\frac{dz}{dt}$  when  $t = 0$ .

8. An observer at  $(3, 6)$  is watching an object descend the graph  $y^2 = x$ . At what point of it's path is the object closest to the observer.

9. Find the area of surface  $\int_S \int z^2 ds$  over the hemisphere  $z = \sqrt{1 - x^2 - y^2}$ .

10. Find the volume of the solid bounded above by the cone  $z = 2 - \sqrt{x^2 + y^2}$  and below by the disk  $R: x^2 + (y - 1)^2 \leq 1$ .