1. For a system described by the following equations (20%)

$$\begin{cases} \dfrac{dx}{dt} - y = 0, x(0) = 0 \\ \dfrac{dy}{dt} + 2x + 3y = 1, y(0) = 0 \end{cases}$$

(a) Find the Laplace transform $X(s)$ of $x(t)$ and $Y(s)$ of $y(t)$. (10%)

(b) Determine the final value of $x(t)$ and $y(t)$ as $t \to \infty$. (5%)

(c) Find the solution $x(t)$ and $y(t)$. (5%)

2. Find the solution of the ordinary differential equation and what are the eigenvalues of the differential equation? (15%)

$$y'' + 5y' + 4y = 3 - 2x, \quad y(0) = \frac{11}{8}, \quad y'(0) = \frac{1}{2}$$

3. Find the first order differential equation and its solution (15%)

$$(2x^3 + 3y)dx + (3x + y - 1)dy = 0$$

4. Find a basis of eigenvectors for the following matrix and diagonalize it. (15%)

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

5. Using Green's theorem, evaluate the line integral $\oint_c \vec{F}(\vec{r}) \cdot d\vec{r}$ counterclockwise around the boundary C of the region R. (15%)

$\vec{F} = [-2y, 2x]$, R is the circle of radius 0.5 about (0, 0)

(Hint: Green's Theorem: $\iint_R (\text{curl}\,\vec{F}) \cdot \vec{k}\, dx\, dy = \oint_c \vec{F}(\vec{r}) \cdot d\vec{r}$)

6. Solve the following partial differential equation (20%)

$$u = u(r, \Phi)$$

$$\nabla^2 u = \frac{\partial}{\partial r}\left(r^2 \frac{\partial u}{\partial r}\right) + \frac{1}{\sin\emptyset}\frac{\partial}{\partial \emptyset}\left(\sin\emptyset \frac{\partial u}{\partial \emptyset}\right) = 0$$

Boundary conditions:

$$u(R, \Phi) = f(\Phi)$$

$$\lim_{r\to\infty} u(r, \emptyset) = 0$$